Area Under The Curves

Question1

The area bounded by the curve $y = \sin\left(\frac{x}{3}\right)$, x axis, the lines $x = 0$ and $x = 3\pi$ is

KCET 2025

Options:

A. 9 sq. units

B. $\frac{1}{3}$ sq. units

C. 6 sq. units

D. 3 sq. units

Answer: A

Solution:

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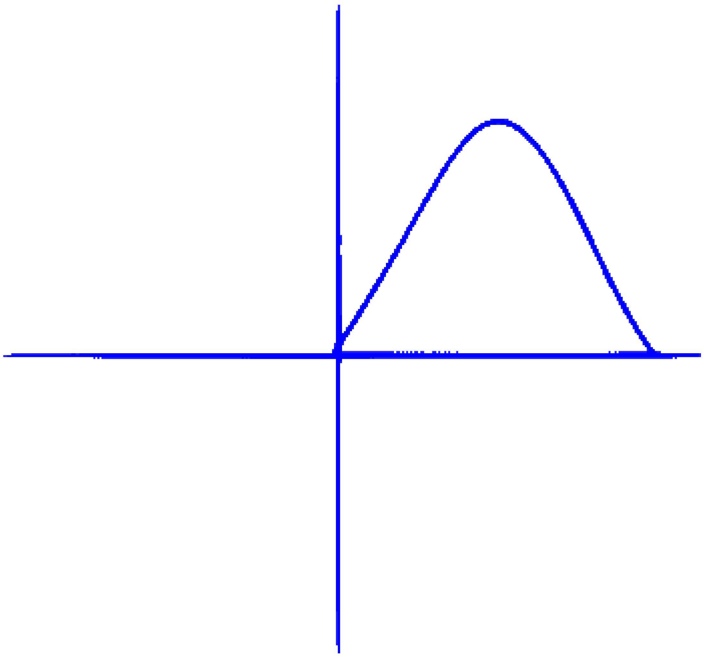

$$\int_0^{3\pi} \sin\left(\frac{x}{3}\right)dx$$
$$\left(-3 \cdot \cos\frac{x}{3}\right)_0^{3\pi} = (-3(-1)) - (-3(1)) = 6$$

Question2

The area of the region bounded by the curve $y = x^2$ and the line $y = 16$ is

KCET 2025

Options:

A. $\frac{32}{3}$ sq. units

B. $\frac{256}{3}$ sq. units

C. $\frac{64}{3}$ sq. units

D. $\frac{128}{3}$ sq. units

Answer: B

Solution:

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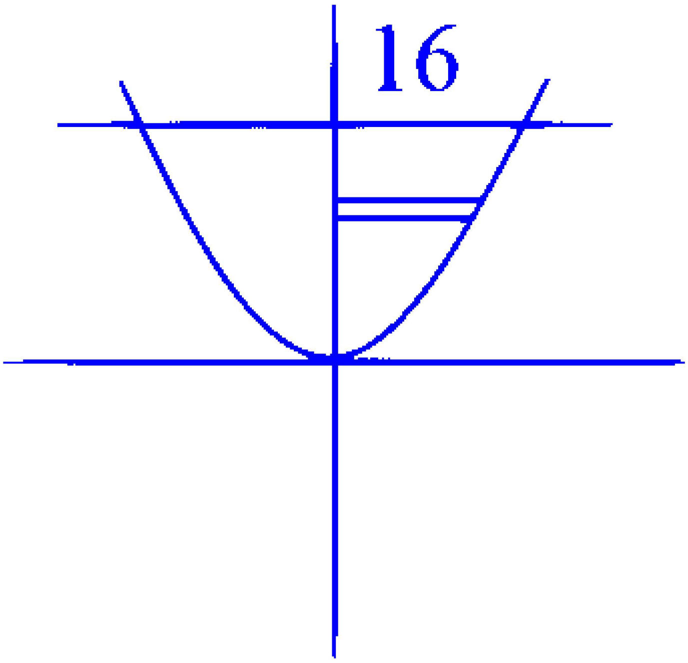

$y=x^2, y=16$

$2\int_0^{16} \sqrt{y} dy=2 \frac{y^{3/2}}{3/2}$

$=\frac{4}{3}\left(4^3\right)=\frac{256}{3}$

Question3

The area of the region bounded by the line $y=3x$ and the curve $y=x^2$ sq units is

KCET 2024

Options:

A. 10

B. 9/2

C. 9

D. 5

Answer: B

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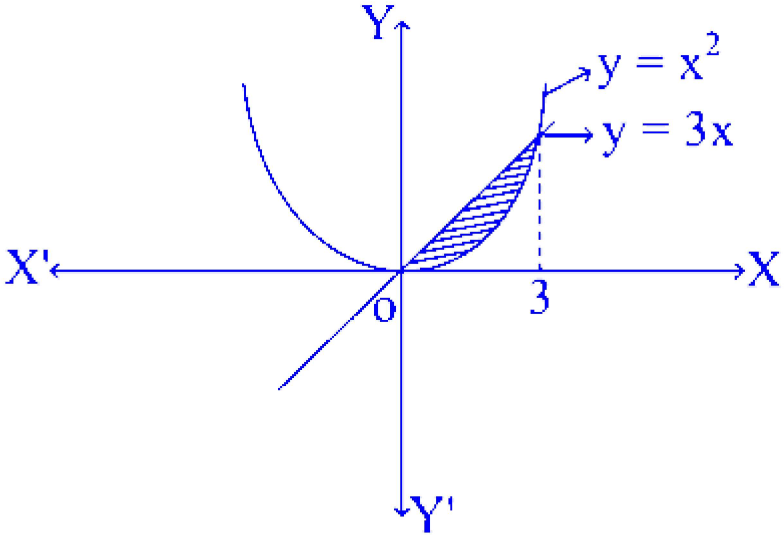
Solution:

We have, $y = 3x$ and $y = x^2 \Rightarrow 3x = x^2$

$x = 3$ and $x = 0$ are the points of intersection.



$$\begin{aligned} \text{Now, area } &= \int_0^3 (3x - x^2)\,dx \\ &= \frac{3x^2}{2}\Big|_0^3 - \frac{x^3}{3}\Big|_0^3 \\ &= 3 \cdot \frac{9}{2} - \frac{27}{3} \\ &= \frac{27}{2} - 9 = \frac{9}{2} \text{ sq units} \end{aligned}$$

Question4

The area of the region bounded by the line $y = x$ and the curve $y = x^3$ is

KCET 2024

Options:

A. 0.2 sq unit

B. 0.3 sq unit

C. 0.4 sq unit

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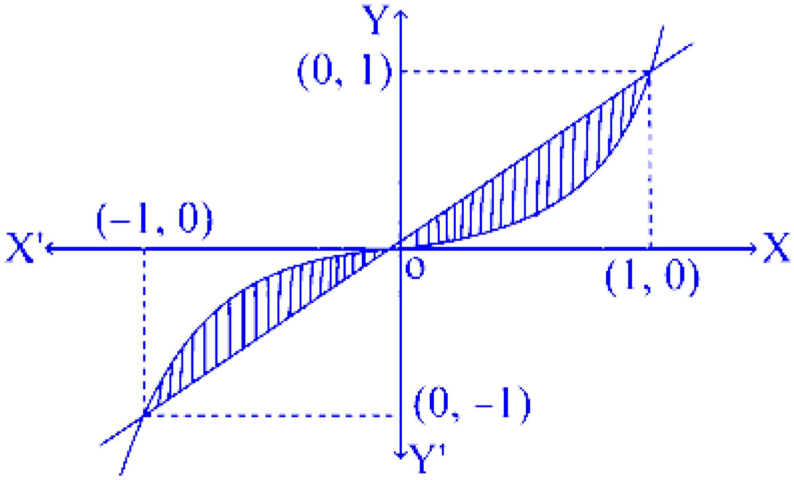
D. 0.5 sq unit

Answer: D

Solution:

$\because y=x$ and $y=x^3$

The curve intersect at $x=-1, y=-1; x=0, y=0$ and $x=1, y=1$.



$$\begin{aligned}\text{Therefore, area } &= -\int_{-1}^{0}\left(x^3-x\right)dx+\int_0^1\left(x-x^3\right)dx \\ &= -\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{-1}^{0}+\left[\frac{x^2}{2}-\frac{x^4}{4}\right]_0^1 \\ &= -\left(\frac{1}{4}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{4}\right) \\ &= -\frac{1}{2}+1=\frac{1}{2}=0.5 \text{ sq. units}\end{aligned}$$

Question5

In the interval $(0, \pi/2)$ area lying between the curves $y=\tan x$ and $y=\cot x$ and the X-axis is

KCET 2023

Options:

A. $2\log 2$ sq units

B. $4 \log 2$ sq units

C. $\log 2$ sq units

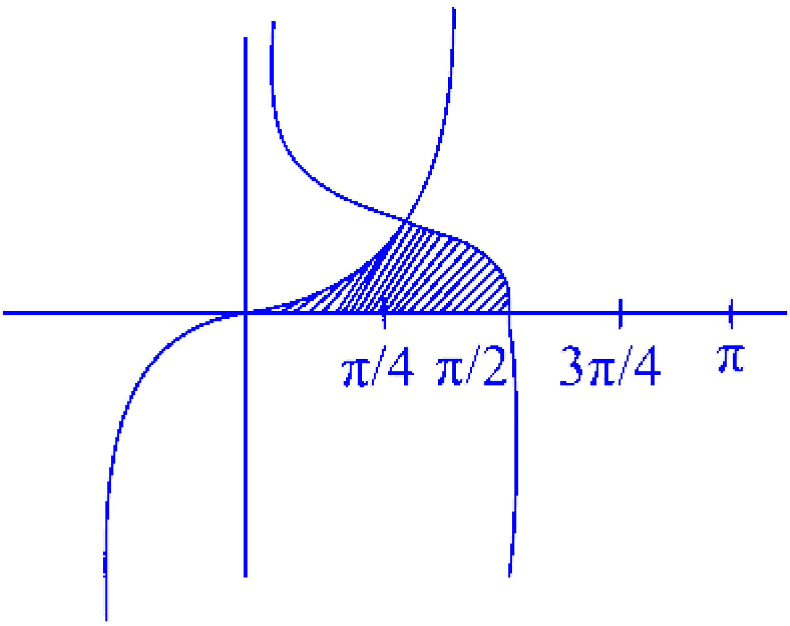
D. $3 \log 2$ sq units

Answer: C

Solution:

Given, $c_1 \rightarrow y = \tan x, x \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

$c_2 \rightarrow y = \cot x \in \left[\frac{\pi}{6}, \frac{\pi}{6}\right]$



The region is symmetric about lines $x = \frac{\pi}{4}$ and its first portion is bounded between the lines $x = 0$ and $x = \frac{\pi}{4}$

$\therefore$ Required Area $= 2 \int\limits_0^{\pi/4} \tan x dy$

$= 2[\log(\sec x)]_0^{\frac{\pi}{4}}$

$= 2\left[\log\left(\sec \frac{\pi}{4}\right) - \log(\sec 0)\right]$

$= 2 \log \sqrt{2} = \log 2$ sq units.

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Question6

The area of the region bounded by the line $y = x + 1$ and the lines $x = 3$ and $x = 5$ is

KCET 2023

Options:

A. $\frac{7}{2}$ sq units

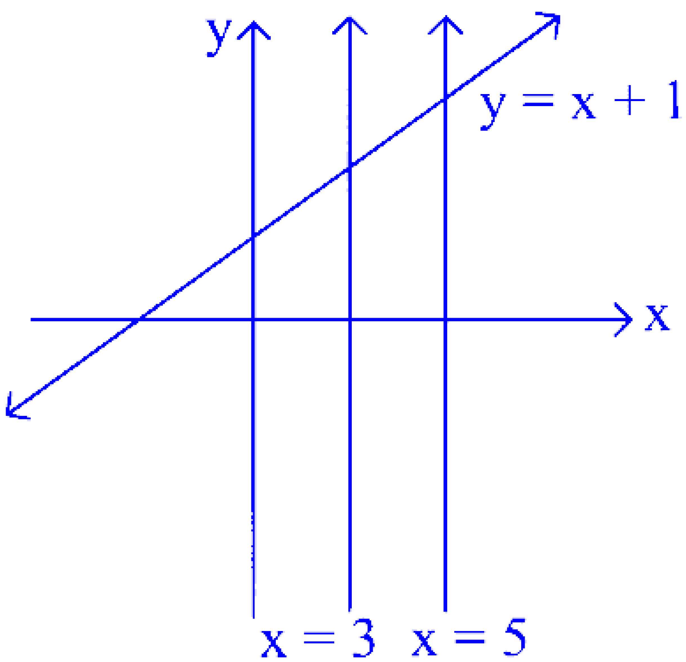
B. $\frac{11}{2}$ sq units

C. 7 sq units

D. 10 sq units

Answer: D

Solution:



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$\therefore$ Required area, $A = \int_3^5 (x+1)dx$

$$= \left[\frac{x^2}{2} + x\right]_3^5 = \left[\frac{25}{2} + 5 - \frac{9}{2} - 3\right]$$

$$= \left[\frac{25 + 10 - 9 - 6}{2}\right] = \frac{20}{2}$$

$= 10$ sq units

Question7

Area of the region bounded by the curve $y = \tan x$, the X-axis and line $x = \frac{\pi}{3}$ is

KCET 2022

Options:

A. $\log \frac{1}{2}$

B. $\log 2$

C. 0

D. $-\log 2$

Answer: B

Solution:

Given, curve $y = \tan x$ and line $x = \frac{\pi}{3}$

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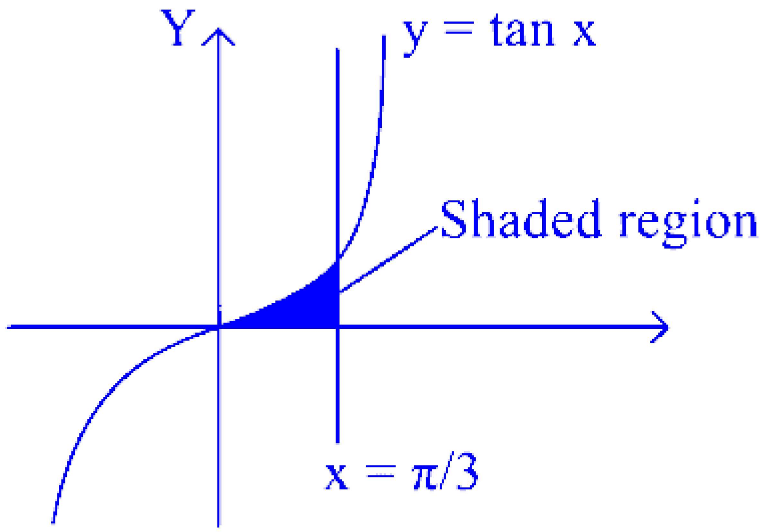

Required area $= \int_0^{\pi/3} \tan x dx = [\log|\sec x|]_0^{\pi/3}$

$= \log\left|\sec\frac{\pi}{3}\right| - \log|\sec 0|$

$= \log(2) - \log(1) = \log 2$ sq units

Question8

The area of the region bounded by $y = -\sqrt{16 - x^2}$ and X-axis is

KCET 2021

Options:

A. 8π sq units

B. 20π sq units

C. 16π sq units

D. 256π sq units

Answer: A

Solution:

The area bounded by $y = \sqrt{16 - x^2}$ and X-axis is shown in the figure.

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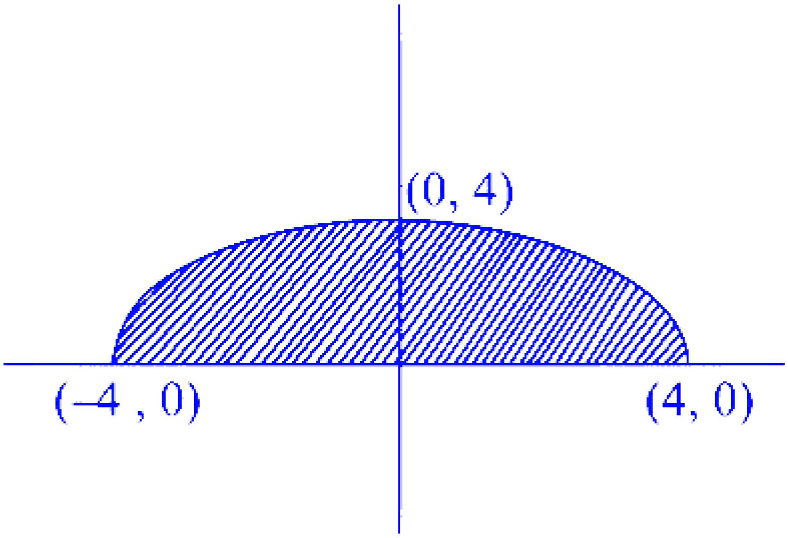

Area $= \int_{-4}^{4} y dx$

$$= \int_{-4}^{4} \sqrt{4^2 - x^2} dx$$

$$= \left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_{-4}^{4}$$

$$= \left[0 + 8\sin^{-1}(1)\right] - \left[\left(8\sin^{-1}(-1)\right)\right]$$

$$= 8\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$$

$$= 8\pi$$

Question9

The area of the region bounded by the curve $y^2 = 8x$ and the line $y = 2x$ is

KCET 2020

Options:

A. $\frac{16}{3}$ sq. units

B. $\frac{4}{3}$ sq. units

C. $\frac{3}{4}$ sq. units

D. $\frac{8}{3}$ sq. units

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Answer: B

Solution:

Given equation of curve,

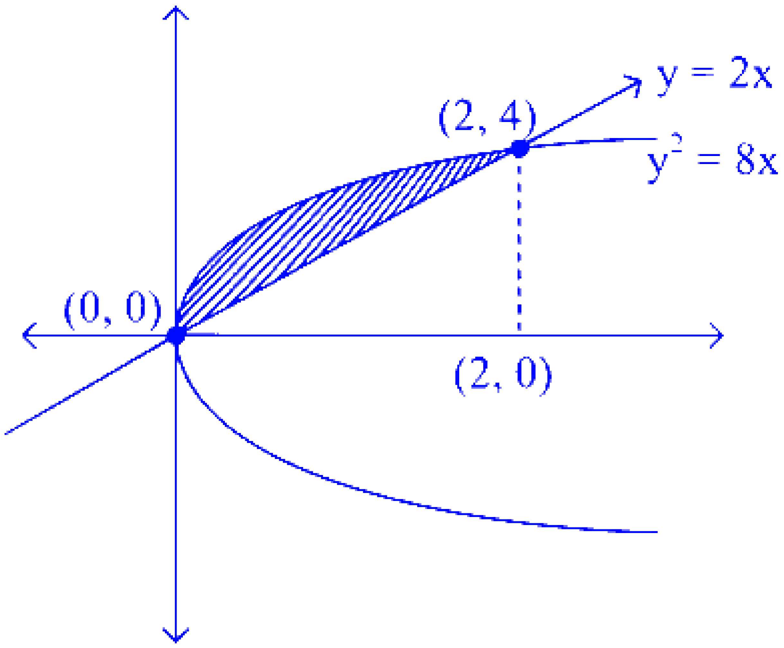
$y^2 = 8x$ and line $y = 2x$

Now, intersecting points of given curve and line.

$$\begin{aligned} (2x)^2 &= 8x \\ \Rightarrow \quad 4x^2 &= 8x \\ \Rightarrow \quad x &= 0, 2 \end{aligned}$$

Putting values of x in $y = 2x$ then, we get $y = 0, 4$

$\therefore$ Required area of bounded region.



$$= \int_0^2 (\sqrt{8x} - 2x)dx$$
$$= \int_0^2 \left(2\sqrt{2} \cdot x^{1/2} - 2x\right)dx$$
$$= 2\int_0^2 \left(\sqrt{2} \cdot x^{1/2} - x\right)dx$$
$$= 2\left[\sqrt{2}\int_0^2 x^{1/2}dx - \int_0^2 xdx\right]$$
$$= 2\left[\sqrt{2}\left(\frac{x^{3/2}}{3/2}\right)_0^2 - \left(\frac{x^2}{2}\right)_0^2\right]$$
$$= 2\left[\sqrt{2} \cdot \frac{2}{3}(2)^{3/2} - 2\right]$$
$$= 2\left[\frac{8}{3} - 2\right] = 2\left(\frac{2}{3}\right) = \frac{4}{3} \text{ sq. units}$$

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Question10

The area of the region bounded by the line $y = 2x + 1$, X-axis and the ordinates $x = -1$ and $x = 1$ is

KCET 2020

Options:

A. $\frac{9}{4}$

B. 2

C. $\frac{5}{2}$

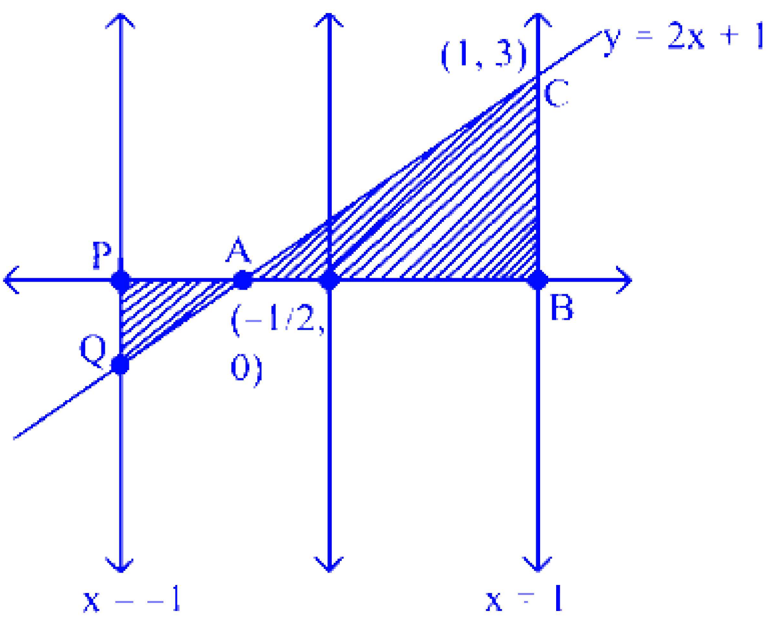
D. 5

Answer: C

Solution:

Given, equation of line $y = 2x + 1$ .... (i)

Eq. (i) passing through the points $\left(\frac{-1}{2}, 0\right)$ and $(0, 1)$.



$\therefore$ Required area of shaded region

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$$= \int_{-1}^{-1/2} -(2x+1)dx + \text{ Area of } \triangle ABC =$$

$$= -\left[2\left(\frac{x^2}{2}\right)+x\right]_{-1}^{-1/2} + \frac{1}{2} \times AB \times BC$$

$$= -\left[x^2+x\right]_{-1}^{-1/2} + \frac{1}{2} \times \frac{3}{2} \times 3$$

$$= -\left[\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - (-1)^2 - (-1)\right] + \frac{9}{4}$$

$$= -\left[\frac{1}{4} - \frac{1}{2}\right] + \frac{9}{4}$$

$$= \frac{1}{4} + \frac{9}{4} = \frac{10}{4} = \frac{5}{2} \text{ sq units}$$

Question11

The area of the region above X-axis included between the parabola $y^2 = x$ and the circle $x^2 + y^2 = 2x$ in square units is

KCET 2019

Options:

A. $\frac{2}{3} - \frac{\pi}{4}$

B. $\frac{\pi}{4} - \frac{3}{2}$

C. $\frac{\pi}{4} - \frac{2}{3}$

D. $\frac{3}{2} - \frac{\pi}{4}$

Answer: C

Solution:

Intersecting points of given parabola $y^2 = x$ and circle $x^2 + y^2 = 2x$

$\Rightarrow \quad x^2 + x = 2x \Rightarrow x(x-1) = 0 \Rightarrow x = 0, 1$

at $x = 0, y = 0$ and at $x = 1, y = \pm 1$

$\therefore$ required area $=$ Area of quadrant of circle with radius 1

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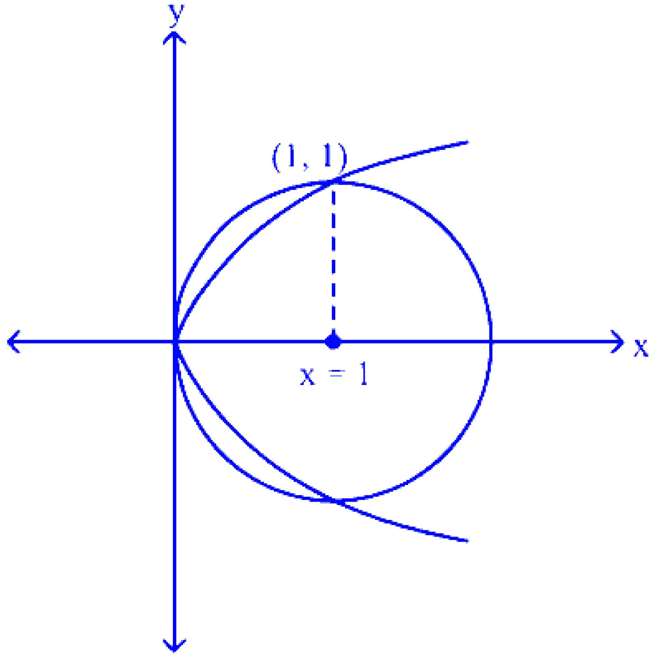

$$\frac{\pi(1)^2}{4} - \int_0^1 \sqrt{x}dx$$

$$= \frac{\pi(1)^2}{4} - \left(\frac{x^{3/2}}{3/2}\right)_0^1 = \frac{\pi}{4} - \frac{2}{3}(1)^3 - 0\Big)$$

$$= \frac{\pi}{4} - \frac{2}{3} \text{ sq units}$$

Question12

The area of the region bounded by Y-axis, $y = \cos x$ and $y = \sin x, 0 \leq x \leq \frac{\pi}{2}$ is

KCET 2019

Options:

A. $\sqrt{2} + 1$ sq units

B. $\sqrt{2} - 1$ sq units

C. $2 - \sqrt{2}$ sq units

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D. $\sqrt{2}$ sq units

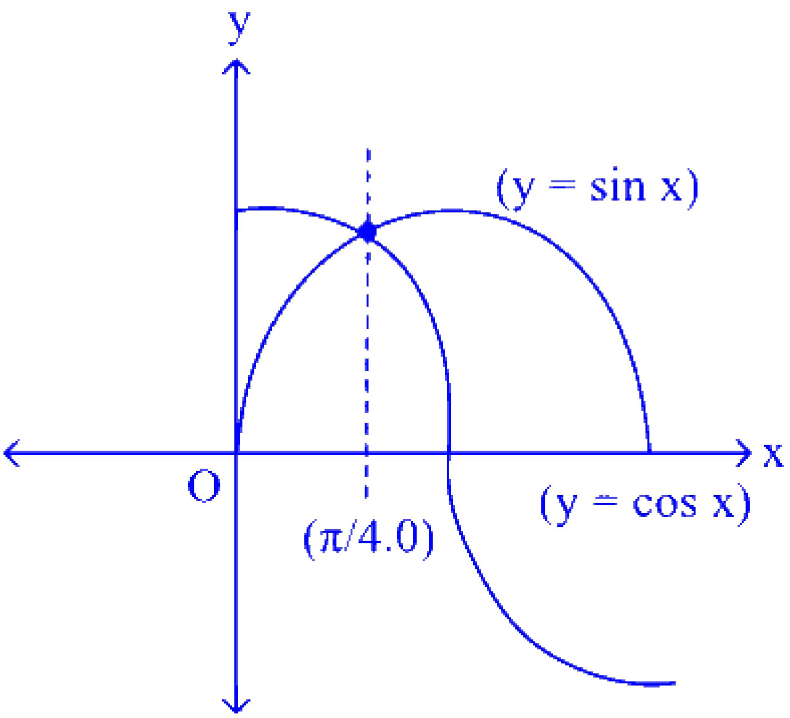
Answer: B

Solution:

For intersecting points of both curve we use $\cos x = \sin x$

$\Rightarrow \quad \tan x = 1 \Rightarrow x = \frac{\pi}{4}$

Required area



$= \int_0^{\pi/4} (\cos x - \sin x) dx$

$$
\begin{aligned}
&= (\sin x + \cos x)_0^{\pi/4} \\
&= \left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right) - (\sin 0 + \cos 0) \\
&= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0 + 1) \\
&= \frac{2}{\sqrt{2}} - 1 = (\sqrt{2} - 1) \text{ sq units}
\end{aligned}
$$

Question13

The area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is

KCET 2018

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Options:

A. 1 sq unit

B. 4 sq units

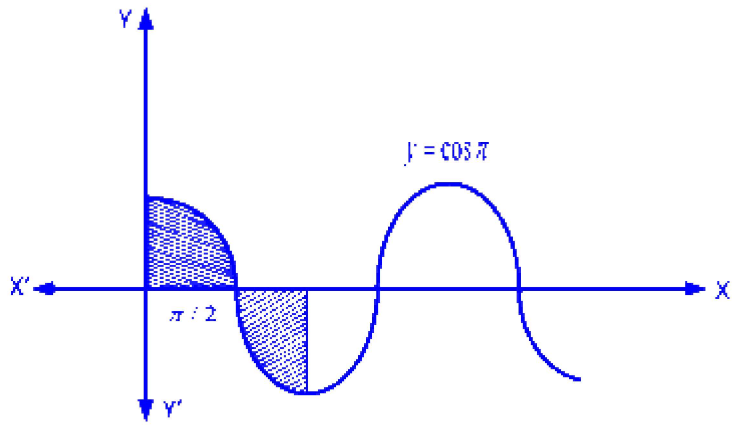
C. 2 sq units

D. 3 sq units

Answer: C

Solution:

Area of region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is shaded region.



Required area of shaded region

$$= \int_0^{\pi/2} \cos x dx + \left| \int_{\pi/2}^{\pi} \cos x dx \right|$$

$$= [\sin x]_0^{\pi/2} + \left| [\sin x]_{\pi/2}^{\pi} \right|$$

$$= \left[\sin \frac{\pi}{2} - \sin 0 \right] + \left| \sin \pi - \sin \frac{\pi}{2} \right|$$

$$= (1 - 0) + |(0 - 1)|$$

$$= 1 + 1 = 2 \text{ sq units}$$

Question14

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The area bounded by the line $y = x$, X-axis and ordinates $x = -1$ and $x = 2$ is

KCET 2018

Options:

A. $\frac{3}{2}$

B. $\frac{5}{2}$

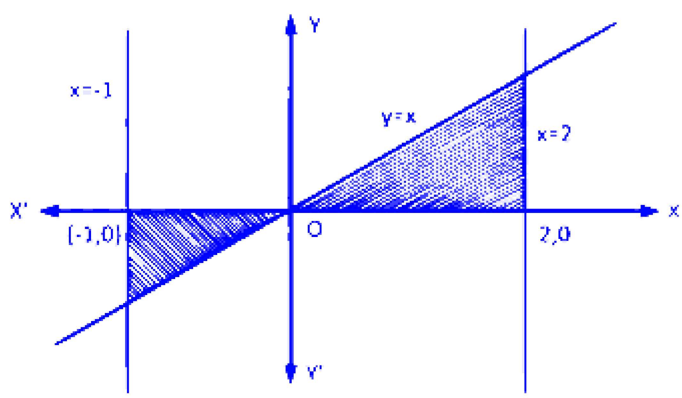
C. 2

D. 3

Answer: B

Solution:

Given line is $y = x$, X-axis, $x = -1$ and $x = 2$



$$\begin{aligned}\text{Required area } &= \left|\int_{-1}^{0} x dx\right| + \int_{0}^{2} x dx \\ &= \left|\left[\frac{x^2}{2}\right]_{-1}^{0}\right| + \left[\frac{x^2}{2}\right]_{0}^{2} \\ &= \left|\left(0 - \frac{1}{2}\right)\right| + (2 - 0) \\ &= \frac{1}{2} + 2 = \frac{5}{2} \text{ sq units}\end{aligned}$$

Question15

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The area of the region bounded by the curve $y = x^2$ and the line $y = 16$

KCET 2017

Options:

A. $\frac{64}{3}$ sq units

B. $\frac{32}{3}$ sq units

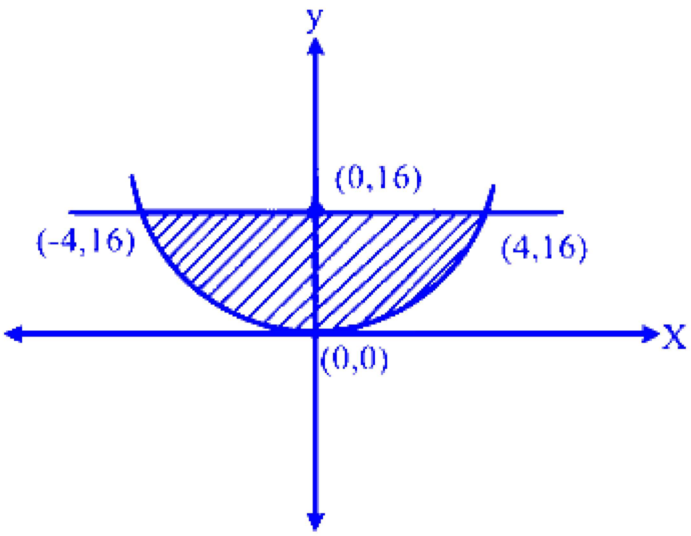
C. $\frac{256}{3}$ sq units

D. $\frac{128}{3}$ sq units

Answer: C

Solution:

According to the question,



$$\begin{aligned}\text{Required area } &= 2\int_0^{16} x\,dy \\ &= 2\int_0^{16} \sqrt{y}\,dy \\ &= 2\left[\frac{y^{3/2}}{3/2}\right]_0^{16} \\ &= \frac{4}{3}\left[16^{3/2} - 0^{3/2}\right] \\ &= \frac{4}{3} \times 64 = \frac{256}{3} \text{ sq units.}\end{aligned}$$

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Question16

Area of the region bounded by the curve $y = \cos x, x = 0$ and $x = \pi$ is

KCET 2017

Options:

A. 4 sq units

B. 3 sq units

C. 1 sq units

D. 2 sq units

Answer: D

Solution:

$$
\begin{aligned}
\textit{Required Area} &= \int_0^{\pi} |\cos x| dx \\
&= 2\int_0^{\pi/2} |\cos x| dx \\
&= 2\int_0^{\pi/2} \cos x dx \\
&= 2[\sin x]_0^{\pi/2} \\
&= 2\left[\sin\frac{\pi}{2} - \sin 0\right] \\
&= 2(1-0) \\
&= 2 \text{ sq units.}
\end{aligned}
$$

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